BIRPASO ARIKETAK

$\int \frac{\sin 3x}{e^x} dx$ ~~16c~~, ~~20d~~, ~~21a~~, ~~23c~~, ~~22a~~, ~~19b~~, ~~17d.~~, ~~19g~~

$$I = \int \frac{\sin 3x}{e^x} dx = \int \underbrace{\sin(3x)}_{S} \cdot \underbrace{e^{-x}}_{E} dx$$

ZATIKAKO METODOA

$$I = \begin{cases} u = e^{-x} \longrightarrow du = -e^{-x} dx \\ dv = \sin 3x \, dx \longrightarrow v = \frac{1}{3}\int 3 \sin(3x)\, dx = -\frac{1}{3}\cos(3x) \end{cases}$$

$$u \cdot v \quad - \int v \cdot du$$

$$I = e^{-x} \cdot \left(-\frac{1}{3}\right) \cdot \cos 3x - \int -\frac{1}{3}\cos(3x) \cdot (-e^{-x})\, dx =$$

$$I = -\frac{1}{3} e^{-x} \cos(3x) - \frac{1}{3} \underbrace{\int \underbrace{\cos(3x)}_{S} \cdot \underbrace{e^{-x}}_{E} \cdot dx}_{I_1} =$$

$$I_1 = \begin{cases} u = e^{-x} \longrightarrow du = -e^{-x} dx \\ dv = \cos 3x \, dx \longrightarrow v = \frac{1}{3}\int 3 \cos 3x \, dx = \frac{1}{3}\sin(3x) \end{cases}$$

$$u \cdot v \quad - \int v \cdot du$$

$$I_1 = e^{-x} \cdot \frac{1}{3}\sin(3x) - \int \frac{1}{3}\sin(3x) \cdot (-e^{-x})\, dx$$

$$I_1 = \frac{1}{3} e^{-x} \sin 3x + \frac{1}{3} \underbrace{\int \sin 3x \cdot e^{-x}\, dx}_{I.}$$

GUZTIRA:

$$I = -\frac{1}{3} e^{-x} \cos(3x) - \frac{1}{3} I_1$$

$$I = -\frac{1}{3} e^{-x} \cos(3x) - \frac{1}{3} \cdot \left[\frac{1}{3} e^{-x} \sin(3x) + \frac{1}{3} \overbrace{\boxed{\int \sin 3x \cdot e^{-x} dx}}^{I} \right.$$

$$I = -\frac{1}{3} e^{-x} \cdot \cos(3x) - \frac{1}{9} e^{-x} \sin(3x) - \frac{1}{9} I$$

$$I + \frac{1}{9} I = -\frac{1}{3} e^{-x} \cos(3x) - \frac{1}{9} e^{-x} \sin(3x)$$

$$\frac{10}{9} I = \frac{-e^{-x}}{9} \left(3\cos 3x + \sin 3x \right)$$

$$\boxed{I = \frac{-e^{x}}{10} \left(3\cos 3x + \sin 3x \right)}$$

24 a)

$$\int \frac{x+4}{\sqrt{1-x^2}} dx = \int \underbrace{\frac{x}{\sqrt{1-x^2}}}_{I_1} + \underbrace{\frac{4}{\sqrt{1-x^2}}}_{I_2} dx$$

$$I_1 = \int \frac{x}{\sqrt{1-x^2}} dx = \int \frac{-dt}{2\sqrt{t}} = -\frac{1}{2}\int t^{-1/2} dt =$$

ordetk met

$$t = 1-x^2$$

$$dt = -2x\, dx$$

$$-\frac{dt}{2} = x\, dx$$

$$= -\frac{1}{2} \frac{t^{1/2}}{1/2} = -\sqrt{t} + k =$$

$$= \boxed{-\sqrt{1-x^2}}$$

$$I_2 = \int \frac{4}{\sqrt{1-x^2}} dx = 4 \arcsin x$$

$$I = I_1 + I_2 = \boxed{-\sqrt{1-x^2} + 4\arcsin x + k}$$

③

Beste bat

$$\int \frac{e^x}{1-\sqrt{e^x}}\,dx = \int \frac{2t\,dt}{1-\sqrt{t^2}} = \int \frac{2t\,dt}{1-t}$$

Ordezkapen metodoa $\left| \begin{array}{l} e^x = t^2 \\ e^x dx = 2t\,dt \end{array} \right.$

$$= 2\int \frac{t\,dt}{1-t} = 2\int -1 + \frac{1}{1-t}\,dt$$

$$\begin{array}{r|l} t & 1-t \\ \hline -t & -1 \\ 0\ +1 & \end{array}$$

$$= 2\left[-\cancel{t} - \ln|1-t|\right] + K.$$

$$\boxed{I = 2\left[-\sqrt{e^x} - \ln|1-\sqrt{e^x}|\right] + K.}$$

22 d

$$\int \frac{\sin x}{\cos^4 x}\,dx = \int \frac{-dt}{t^4} = -\frac{t^{-4+1}}{-4+1} + K$$

Ordezkapena

$t = \cos x$

$dt = -\sin x\,dx$

$$= \frac{+1}{+3t^3} + K$$

$$= \boxed{\frac{1}{3\cos^3 x} + K}$$

21 a

$$\int \frac{\ln\sqrt{x}}{\sqrt{x}}\,dx = \int \underset{P}{x^{-1/2}} \cdot \underset{L}{\ln\sqrt{x}}\,dx.$$

Zatikako metodoa

$$\left| \begin{array}{l} u = \ln\sqrt{x} \to du = \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2x}\,dx \\ dv = x^{-1/2}dx \to v = \frac{x^{-1/2+1}}{-1/2+1} = \frac{1}{1/2}\sqrt{x} = 2\sqrt{x} \end{array} \right.$$

$$\int \frac{\ln\sqrt{x}}{\sqrt{x}}\,dx = \underset{u \cdot v}{\ln\sqrt{x} \cdot 2\sqrt{x}} - \underset{-\int v \cdot du}{\int \cancel{2}\sqrt{x} \cdot \frac{1}{\cancel{2}x}\,dx} =$$

$$= 2\sqrt{x} \cdot \ln\sqrt{x} - \int x^{-1/2}dx = 2\sqrt{x} \cdot \ln\sqrt{x} - 2\sqrt{x} + K$$

$$\frac{x^{-1/2+1}}{-1/2+1} = \frac{\sqrt{x}}{1/2} = 2\sqrt{x}$$

$$\boxed{= 2\sqrt{x} \cdot (\ln\sqrt{x} - 1) + K}$$

④

21b $\displaystyle\int \frac{1-\sin x}{x+\cos x}\,dx$

Berechnen $\longrightarrow$ $\boxed{\ln|x+\cos x|+k}$

26 c $\displaystyle I=\int \frac{x^4-2x-6}{x^3+x^2-2x}\,dx$

$$\begin{array}{l|l} x^4 \quad -2x-6 & x^3+x^2-2x \\ -x^4-x^3+2x^2 & x-1 \\ \hline /\ -x^3+2x^2-2x-6 & \\ +x^3+x^2-2x & \\ \hline /\ 3x^2-4x-6 & \end{array}$$

$$I=\int x-1+\underbrace{\frac{3x^2-4x-6}{x^3+x^2-2x}}_{I_1}\,dx.$$

$$I_1=\int \frac{3x^2-4x-6}{x^3+x^2-2x}\,dx=\int \frac{3x^2-4x-6}{x(x+2)(x-1)}\,dx.$$

$$\frac{3x^2-4x-6}{x(x+2)(x-1)}=\frac{A}{x}+\frac{B}{x+2}+\frac{C}{x-1}$$

$$3x^2-4x-6=A(x+2)(x-1)+B\cdot x(x-1)+C(x+2)x.$$

$x=0 \longrightarrow -6=-2A+\cancel{B\cdot 0}+\cancel{C\cdot 0} \longrightarrow \boxed{A=3}$

$x=1 \longrightarrow -7=\cancel{A\cdot 0}+\cancel{B\cdot 0}+3C \longrightarrow \boxed{C=-7/3}$

$x=-2 \longrightarrow 14=\cancel{A\cdot 0}+6B+\cancel{C\cdot 0} \longrightarrow \boxed{B=7/3}$

$$I_1 = \int \frac{3}{x} + \frac{7/3}{x+2} + \frac{-7/3}{x-1}\, dx$$

$$I_1 = 3\cdot \ln|x| + \frac{7}{3}\ln|x+2| - \frac{7}{3}\ln|x-1|$$

$$I_1 = \ln\left|\frac{x^3(x+2)}{(x-1)}\right|$$

$$I = \int\left(x-1 + \frac{3}{x} + \frac{7/3}{x+2} + \frac{-7/3}{x-1}\right)dx =$$

$$= \boxed{\frac{x^2}{2} - x + \ln\left|\frac{x^3(x+2)}{(x-1)}\right| + k}$$

edo $\boxed{\frac{x^2}{2} - x + 3\ln|x| + \frac{7}{3}\ln|x+2| - \frac{7}{3}\ln|x-1| + k}$

17 c

$$\int \frac{\sqrt{x}\,dx}{\sqrt[3]{x}-1}$$

6.

ORDEZKAPEN METODOA
Aldagaia biltzeko
erroaren mkt → $\boxed{6}$

$$\int \frac{\sqrt{u^6}\;6\cdot u^5\,du}{\sqrt[3]{u^6}-1} =$$

$$\begin{cases} x = u^6 \\ dx = 6u^5\,du \end{cases}$$

$$\int \frac{u^3\cdot 6u^5\,du}{u^2-1} = \int \frac{6u^8\,du}{u^2-1}$$ → Fu arrazionala P(x)ren maila > Q(x) rena baita.

$$\begin{array}{l|l} u^8 & u^2-1 \\ \cline{2-2} -u^8 \quad +u^6 & u^6+u^4+u^2+1 \\ \hline \quad u^6 & \\ \quad -u^6 \quad +u^4 & \\ \hline \qquad u^4 & \\ \qquad -u^4 \quad +u^2 & \\ \hline \qquad\quad u^2 & \\ \qquad\quad -u^2 \quad +1 & \\ \hline \qquad\qquad 1 & \end{array}$$

$$I = 6\int u^6+u^4+u^2+1+\underbrace{\frac{1}{u^2-1}}_{I_1}\,dx.$$

→ Fu arrazionala erro bakunak

$$I_1 = \frac{1}{u^2-1} = \frac{1}{(u+1)(u-1)} = \frac{A}{u+1}+\frac{B}{u-1}$$

$$1 = A(u-1)+B(u+1)$$

$u=1 \longrightarrow 1 = A\cdot\cancel{0} + B\cdot 2 \longrightarrow \boxed{B=1/2}$

$u=-1 \longrightarrow 1 = -2A + B\cdot\cancel{0} \longrightarrow \boxed{A=-1/2}$

$$I_1 = \int \frac{-1/2}{u+1} + \frac{1/2}{u-1}\, du$$

$$I = 6 \int \left(u^6 + u^4 + u^2 + 1 + \frac{-1/2}{u+1} + \frac{1/2}{u-1} \right) du$$

$$= 6 \cdot \frac{u^7}{7} + \frac{u^5}{5} + \frac{u^3}{3} + u - \frac{1}{2} \ln|u+1| + \frac{1}{2} \ln|u-1| + k$$

Desegin aldaketa $\boxed{x = u^6 \longrightarrow u = \sqrt[6]{x} = x^{1/6}}$

$$I = 6 \left[\frac{x^{7/6}}{7} + \frac{x^{5/6}}{5} + \frac{x^{3/6}}{3} + x^{1/6} - \frac{1}{2} \ln|\sqrt[6]{x}+1| + \frac{1}{2} \ln|\sqrt[6]{x}-1| + k \right]$$

$$\boxed{= 6 \left[\frac{x\sqrt[6]{x}}{7} + \frac{\sqrt[6]{x^5}}{5} + \frac{\sqrt{x}}{3} + \sqrt[6]{x} + \frac{1}{2} \ln\left| \frac{\sqrt[6]{x}-1}{\sqrt[6]{x}+1} \right| + k \right.}$$